

2019 WMTTC

少年组个人赛第一轮

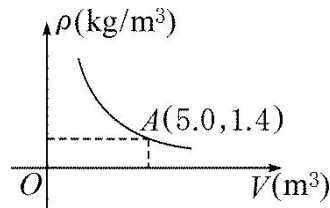
Intermediate Level Individual Round 1

1. Symbol $[x]$ represents the largest integer up to x , then

$$\left[\frac{\sin 45^\circ}{\cos 45^\circ - \cos 30^\circ} \right] = \underline{\hspace{2cm}}.$$

2. If $\frac{1}{\sqrt{4}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{7}} + \cdots + \frac{1}{\sqrt{n}+\sqrt{n+1}} = 8$, then $n = \underline{\hspace{2cm}}$.

3. Known in the coordinate system ρ - O - V , the equation of the curve is $\rho = \frac{m}{V}$. If point $(5.0, 1.4)$ on the curve, then $m = \underline{\hspace{2cm}}$.

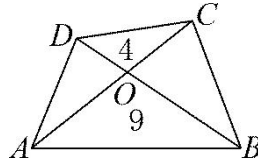


4. If $\sqrt{x-5} + |5-2x| = 2x+1$, then $x = \underline{\hspace{2cm}}$.

5. If $\frac{a_1 + a_2 + a_3 + a_4 + a_5 + a_6}{6} = 20$, $\frac{b_1 + b_2 + b_3}{3} = 60$, then

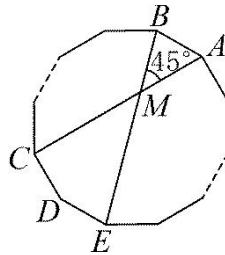
$$\frac{a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + b_1 + b_2 + b_3}{3} = \underline{\hspace{2cm}}.$$

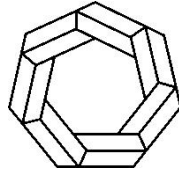
6. Known $ABCD$ is a quadrilateral, AC and BD intersect at point O , the area of $\triangle AOB$ is 9, and the area of $\triangle COD$ is 4. Then the minimum area of quadrilateral $ABCD$ is _____.



7. The equation $\frac{c+2}{2} \cdot x^2 - (c+2)x + c+1 = 0$ in terms of x has real roots, then the probability that this equation has a negative root is _____.

8. As shown in the figure, let AB , CD , DE be three sides of the regular n -polygon, AC and BE intersect at point M . If $\angle AMB = 45^\circ$, then $n =$ _____.



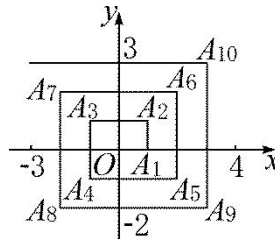


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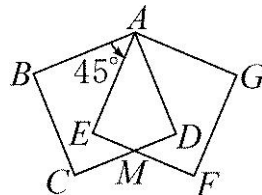
少年组个人赛第二轮

Intermediate Level Individual Round 2

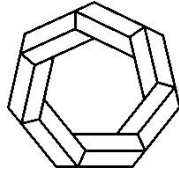
9. The solution set of x for the inequalities system $\begin{cases} x - 3m < 0 \\ n - 2x < 0 \end{cases}$ is $-1 < x < 3$, then the value of $(m+n)^{2019}$ is _____.
10. As shown in the figure, $O(0,0)$, $A_1(1,0)$, $A_2(1,1)$, $A_3(-1,1)$, $A_4(-1,-1)$, $A_5(2,-1)$, \dots . If the length of $OA_1A_2A_3\dots A_{n-1}A_nM$ is 2019, then the coordinates for point M are _____.



11. As shown in the figure, the square $ABCD$ rotates around point A 45° and becomes square $AEFG$, CD and EF intersect at point M . If $DM=5$, then $CM=$ _____.



12. The equation $x^2 - 10x - 9n^2 + 36n = 0$ in terms of x has two integer roots, then the value of positive integer n is _____.

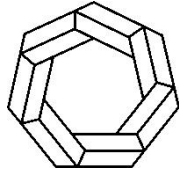


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Intermediate Level Individual Round 3

13. If n is a natural number, and $2^{10} + 2^{13} + 2^n$ is a square number, then $n =$ _____.
14. The roots of equation $(7-k)(8-k)x^2 - (112-15k)x + 56 = 0$ in terms of x are integers. How many values of integer k ? Answer: _____.



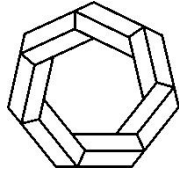
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Intermediate Level Relay Round 1

1-A

Known $x, y, z > 0$, $xyz=1$, then the minimum value of $x + y + z$ is _____.



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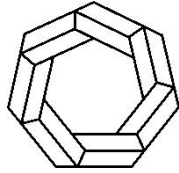
Intermediate Level Relay Round 1

1-B

Let \mathbf{T} be the number you will receive.

If $x_i = 0$, or 1, or -2 ($i=1, 2, 3, \dots, n$) and
$$\begin{cases} x_1 + x_2 + \dots + x_n = -\mathbf{T}, \\ x_1^2 + x_2^2 + \dots + x_n^2 = 673 \times \mathbf{T}, \end{cases}$$

then the value of $x_1^3 + x_2^3 + \dots + x_n^3$ is _____.



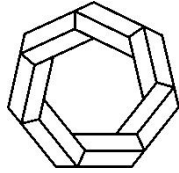
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Intermediate Level Relay Round 2

2-A

The roots of equation $ax^2 + 2(2a-1)x + 4a - 7 = 0$ in terms of x are integers. How many values of a ? Answer: _____.



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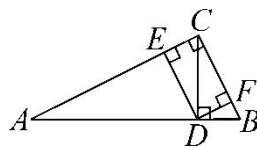
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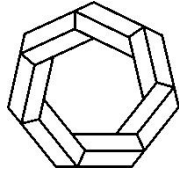
Intermediate Level Relay Round 2

2-B

Let T be the number you will receive.

Known in $\triangle ABC$, $\angle ACB = \angle AED = \angle ADC = \angle DFB = 90^\circ$, $AC = 2BC$,
and $BF = T$, then $AE =$ _____.





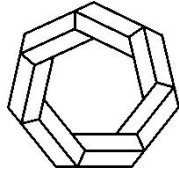
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Intermediate Level Relay Round 3

3-A

Take three numbers from 2,3,5,7 to form 3-digit number \overline{abc} , if \overline{abc} is divisible by $a \times b \times c$, then $a =$ _____.



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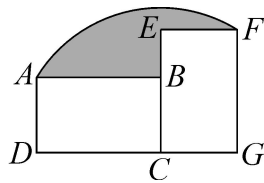
少年组接力赛第三轮

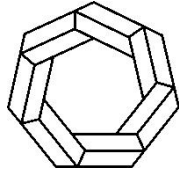
Intermediate Level Relay Round 3

3-B

Let T be the number you will receive.

As shown in the figure, the rectangle $ABCD$ rotates around point C 90° and becomes rectangle $FGCE$, and the area of shadow is T , the area of rectangle $ABCD$ is $T + 1$, then the length of DG is _____. ($\pi=3$)





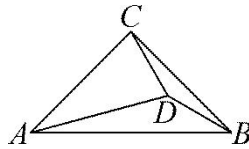
2019 WMTTC

少年组团体赛

Intermediate Level Team Round

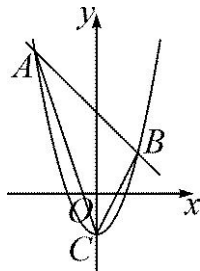
1. If $3^a = 5^b = m$, $\frac{1}{a} + \frac{1}{b} = 2$, then the value of $m =$ _____.

2. As shown in the figure, in $\triangle ABC$, $AC = CB = AD$, $BD = DC$, $\angle DBA = 2\angle DAB$, $\angle DAC = 2\angle DBC$, then $\angle ADC =$ _____ $^\circ$.



3. If $\frac{ab}{a+b} = 2$, $\frac{bc}{b+c} = 3$, $\frac{ca}{c+a} = 4$, then the value of $a =$ _____.

4. As shown in the figure, it is known that parabola $y = x^2 - 2$ and straight line $y = kx + b$ intersect points $A(-3, 7)$ and $B(2, 2)$, intersect y -axis at point C , then the area of $\triangle ABC$ is _____.



5. If $[x]$ represents the largest integer which is no more than x , and $[x+1] = 2x - 3$, then the value of $x =$ _____.

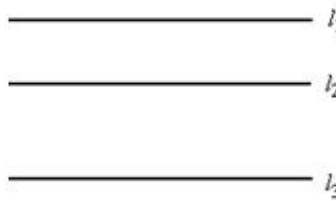
6. Known that x, m, n are natural numbers, and $x+178 = m^3$, $x-291 = n^3$, then the value of $x =$ _____.

7. Known that the three sides of $\triangle ABC$ are a, b, c ,

and $b = \sqrt{\frac{a-2}{5a-4}} + \sqrt{\frac{a-2}{4-5a}} + a$, then the maximum area of $\triangle ABC$ is _____.

8. There are _____ pairs of positive integers (x, y) that satisfy $xy = 2019(x+y)$.

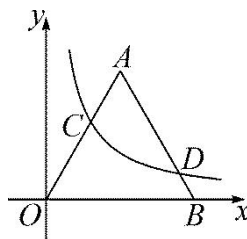
9. As shown in the figure, l_1, l_2, l_3 are straight lines, and $l_1 \parallel l_2 \parallel l_3$. If the distance between l_1 and l_2 is 5, the distance between l_2 and l_3 is 7, and a square has three vertices on three straight lines, then the maximum area of this square is _____.



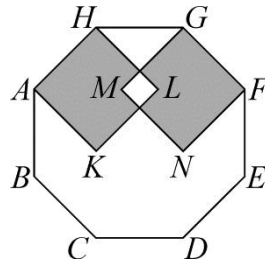
10. If a, b, c, d are continuous natural numbers, and $a^3 + b^3 + c^3 = d^3$, then the value of $a + b + c + d =$ _____.

11. If α, β are rational numbers, and α, β are two roots of equation $(m+1)x^2 - (m-1)x - 10 = 0$ in terms of x , and $(\alpha+1)(\beta+1) = m-4$, then the value of $\alpha^2 + \beta^2$ is _____.

12. As shown in the figure, the hyperbola $y = \frac{k}{x} (x > 0)$ and the edge OA and AB of the equilateral $\triangle AOB$ intersect at C and D , respectively. If $OA = 6, OC = 3BD$, then $k =$ _____.



13. $ABCDEFGH$ is a regular octagon, $AKLH$ and $GMNF$ are squares, the area of shadow is $5 - 3\sqrt{2}$, then the area of $ABCDEFGH$ is_____.



14. If the quadratic function $f(x)=ax^2+bx+c$ ($a \neq 0$) satisfies:

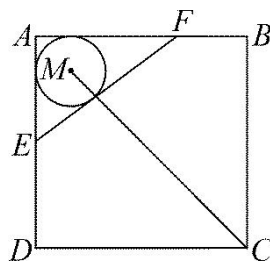
(1) $f(-1)=0$; (2) $x \leq f(x) \leq \frac{x^2+1}{2}$.

Then the minimum value of $f(x)$ is_____.

15. If $a, b, c > 0$ and $a+b+c = \sqrt[3]{4}$, then the minimum value of $\sqrt[3]{a^3+b^3} + \sqrt[3]{b^3+c^3} + \sqrt[3]{c^3+a^3}$ is_____.

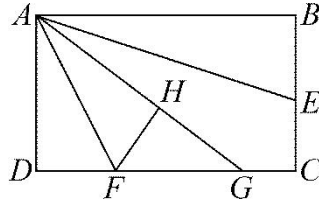
16. In the coordinate system xOy , $A(-1,2)$, $B(1,4)$, point P is on the x -axis, then $\angle APB$ has the maximum value when the coordinate of the point P is_____.

17. As shown in the figure, $ABCD$ is a square, $AB=4$, $AE=ED$, $AF=2FB$, and point M is the inward heart of $\triangle EAF$, then $MC=$ _____.



18. If a and c are natural numbers, $c < a$, $\frac{c}{a} = 0.\underbrace{\dots\dots}_{k}15\dots\dots$, k is an integer, then the minimum value of a is_____.

19. As shown in the figure, in rectangle $ABCD$, point E is on BC , point B symmetry point G about AE on CD , point F is on CD , and point D symmetry point H about AF on AG . If $AB=5$, $BC=3$, then the value of $CE+FH$ is_____.



20. Known that a, b, q, r are positive integers, then there are_____ pairs of (a, b) that satisfy
$$\begin{cases} a^2 + b^2 = (a+b)q + r, \\ q^2 + r = 1831. \end{cases}$$

2019WMTIC Intermediate Level

Individual Rounds

1	2	3	4	5	6	7
-5	99	7	41	100	25	$\frac{1}{2}$
8	9	10	11	12	13	14
12	-1	(17,-22)	$5\sqrt{2}$	4	14	4

Relay Rounds

1-B	2-B	3-B
-2025	24	6

Team Round

1	2	3	4	5	6	7	8	9	10
$\sqrt{15}$	75	$\frac{24}{5}$	15	$\frac{7}{2}$ or 4	2019	2	9	193	18
11	12	13	14	15	16	17	18	19	20
$\frac{61}{9}$	$\frac{81\sqrt{3}}{25}$	2	0	2	(1,0)	$\frac{10\sqrt{2}}{3}$	13	$\frac{17}{6}$	8