2019 WMTC

青年组个人赛第一轮

Advanced Level Individual Round 1

1. The minimum value of $\sqrt{x}-\sqrt{4-x}$ is ______.

2. Known set $P=\{(x,y)|(x-\cos\theta)^2+|y-\sin\theta-2|=0,\theta\in\mathbf{R}\}$, $Q=\{(x,y)|y=kx+5,k<0\}$. If $P\cap Q$ only one element, then $k=$______.

3. If $\{a_n\}$ is a geometric sequence, $a_3=\sqrt{5}$, $a_4=\sqrt[3]{7}$, then $\frac{a_1+a_{2014}}{a_7+a_{2020}}$ =______.

4. Suppose x>0, and $\left|x-\frac{1}{2}\right|+\left|x-\frac{1}{3}\right|+\left|x-\frac{1}{6}\right|=1$, then x=______.

5. Let $[x]$ be the largest integer that is not larger than x. Given a sequence $\{a_n\}$ where $a_1=7$ and $a_{n+1}=\left[a_n+\sqrt{a_n^2-3}\right]$, then the last digit of $a_1+a_2+a_3+\cdots+a_{2019}$ is ______.

6. Suppose $P(x,y)$ is a point on the line $x+3y-9=0$, then the minimum value of $2\cdot 3^x+9^y$ is ______.

7. Divide the odd numbers 1, 3, 5, 7, ⋯ into many groups so that the n^{th} group has $2n$-1 numbers: {1}，{3，5，7}，{9，11，13，15，17}，⋯, then 2019 is in group ______.

8. It is known that the line $l: mx+y+3m-\sqrt{3}=0$ and the circle $P: x^2+y^2=16$ intersect at points *A* and *B*, $AC\perp l$, $BD\perp l$, and *C* and *D* are points on the *x*-axis. If $AB=2\sqrt{7}$, then *CD*=______.

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2019 WMTC

青年组个人赛第二轮

Advanced Level Individual Round 2

9．Known arithmetic sequence $\{a_n\}$ satisfy $a_4=7$, and $S_6=36$, so the sum of the first 2019 terms of the sequence $\left\{\frac{1}{a_n a_{n+1}}\right\}$ is ______.

10． If a and b are positive integers, and $\begin{cases}2ab+b^2=45,\\ ab^2=54,\end{cases}$ then $a+b=$______.

11．If $a, b, c, d, \lambda \in \mathbf{R}^+$, $a^2+b^2+c^2=d^2$, and $a^4+b^4+c^4+d^4 \geq \lambda abcd$, then the maximum value of λ is ______.

12．The focus of parabola E: $x^2=2py(p>0)$ is F, and F is on line l: y=2x+1. Point M is on E, point M and point N are symmetrical about l, if point N on ray $y=6\ (x\leq 4)$, then the chord length of FM cut E is ______.

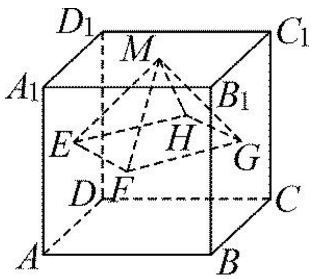
2019 WMTC

青年组个人赛第三轮

Advanced Level Individual Round 3

13．Known the equal ratio sequence $a_{k_1}, a_{k_2}, a_{k_3}, \cdots, a_{k_n}, \cdots$ is a part of the equal difference sequence $\{a_n\}(a_1 \neq a_2)$, and $k_1 = 1, k_2 = 5, k_3 = 21$, then $3k_{2019} + 1 =$_______.

14． The volume of the circumscribed sphere of the cube $ABCD - A_1B_1C_1D_1$ is $\frac{\sqrt{3}}{2}\pi$, as shown in the figure, points *E*, *F*, *G*, *H*, *M* are the centers of the planes in which they are located, then radius of the inscribed sphere of pyramid *M*-*EFGH* is______.



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2019 WMTC

青年组接力赛第一轮

Advanced Level Relay Round 1

1-A

If $f(x)=\log_2\frac{\sqrt{2}x}{1-x}$, $S(n)=f(\frac{1}{n})+f(\frac{2}{n})+$ 鬃 $f(\frac{n-1}{n})$， then $S(101)=$________.

4

2019 WMTC

青年组接力赛第一轮

Advanced Level Relay Round 1

1-B

Let **T** be the number you will receive.

Known sequence $\{a_n\}$ satisfies $a_{n+1}-a_n^2+a_n=1$, and $a_{2020}-a_1=$ **T**, then $(a_1-1)^2+(a_2-1)^2+\cdots+(a_{2019}-1)^2=$________.

5

2019 WMTC

青年组接力赛第二轮

Advanced Level Relay Round 2

2-A

If the positive real number a and b satisfy the equation $3a+b=4$, then the maximum value of $a(a+b)$ is_______.

2019 WMTC

青年组接力赛第二轮

Advanced Level Relay Round 2

2-B

Let **T** be the number you will receive.

Known in $\triangle ABC$, AB=**T**, AC=2BC, then the maximum area of $\triangle ABC$ is _______.

6

2019 WMTC

青年组接力赛第三轮

Advanced Level Relay Round 3

3-A

Solve equation $(x-1)^3+(x-3)^3+(x-5)^3+(x-7)^3=0$， $x=$________.

2019 WMTC

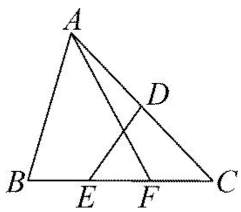
青年组接力赛第三轮

Advanced Level Relay Round 3

3-B

Let **T** be the number you will receive.

As shown in the figure. In $\triangle ABC$, $AB=3$, $AC=\mathbf{T}$, $\angle BAC=60°$, $AD=DC$, $BE=EF=FC$, then the value of $\overrightarrow{DE}\cdot\overrightarrow{AF}=$________.



7

2019 WMTC

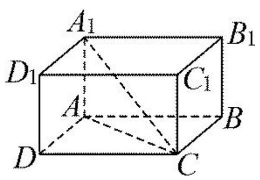
青年组团体赛

Advanced Level Team Round

1． Known $\{a_n\}$ is geometric sequence, and $a_1 = 8$, $q = \frac{1}{2}$. If take any two of its first five terms, then the probability of only one integer is_______.

2． Known sequence $\{a_n\}$ satisfies $a_1 = 1, a_{n+1} = a_n + 2\sqrt{a_n - 1} + 1$, then $a_{11} =$________.

3． As shown in figure, in rectangular box $ABCD$- $A_1B_1C_1D_1$, A_1C=13, AC=5, then the maximum surface area of $ABCD$- $A_1B_1C_1D_1$ is_______.



4． If x, y, z>0, and $xyz(x+y+z)$=9, then the minimum value of $(x+y)^2+4(y+z)^2+(z+x)^2$ is_______.

5． Definition: $u(n)$ represents the units digit of natural number n, example $u(2019)=9$. If $a_n = u(3n) - u(n)$, then $\sum_{i=1}^{2019} a_i =$_______.

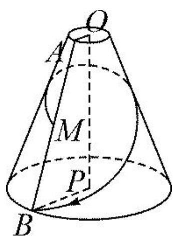
6． If m is a natural number, then there are_________ pairs of prime numbers (p, q) that satisfy $p^2+11pq+25q^2=m^2$.

7. Known $f(x)=2x^3+5x^5$ ($x\in\mathbf{R}$), $g(x)=f(x-7)$, $\{a_n\}$ is an arithmetic sequence with common difference $d\neq 0$. If $g(a_1)+g(a_{11})=0$, then $a_1+a_2+\cdots+a_{11}=$_______.

8. Known the area of $\triangle ABC$ is $\frac{1}{4}$, and a, b, c are the lengths of $\triangle ABC$'s three sides. Then $2b+c$ has the minimum value when $AB+BC+CA=$_______.

9. Known $a>0$, $b>0$, $c>0$, $ab+bc+ca=45$, and $abc=50$, then the minimum value of $a+b+c$ is_______.

10. As shown in the figure. This is a frustum of a cone, upper radius $OA=1$, bottom radius $PB=4$, bus bar $AB=18$, point M is midpoint of AB. If an ant starts from point M make a circle on the side of the circular platform to point B, then the minimum distance it takes is _______.



11. Known n is a positive integer, and $a_1+a_2+\cdots+a_n=n^3$.

Then $\frac{1}{a_2-1}+\frac{1}{a_3-1}+\cdots+\frac{1}{a_{2020}-1}=$_______.

12. If the inequality $[(a-3)x-3](3x^2-ax-3)\geq 0$ holds for any positive number x, then $a=$_______.

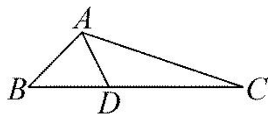
13. If m is a natural number, a, b, c are positive integers, and

$\begin{cases} b-c=m^2, \\ b^2=ac, \\ \log_{12} a+\log_{12} b+\log_{12} c=6. \end{cases}$ Then the maximum value of $a+b+c$ is_______.

14． If the tangent of hyperbola $xy=5$ and hyperbola $xy=3$ intersect at points A and B, then the minimum value of $|AB|$ is_______.

15． Known in $\triangle ABC$, $AB=BC=CA=3$, point E is on BC, point F is on AE, extend BF intersects AC at point D. If $BE=1$, $\overrightarrow{AD}=\lambda\overrightarrow{DC}$, so $\left|\overrightarrow{CF}\right|$ has the minimum value when $\lambda=$_______.

16． As shown in the figure. In $\triangle ABC$, point D is on BC, $AB=2\sqrt{2}$, $BD=3$, $\cos\angle BAD=\frac{\sqrt{10}}{10}$, $\angle DAC=\frac{\pi}{4}$. Then the area of $\triangle ABC$ is _______.



17． If $\theta\in\left(0,\frac{\pi}{2}\right)$, then the minimum value of $\frac{(\tan\theta+\cot\theta)^4}{\sqrt[3]{\tan^3\theta+\cot^3\theta}}$ is_______.

18． Known $f(x)=\cos\pi x+2x-\frac{e^2}{2x-1}-1$, if $f(x_1)=f(x_2)=0$, then $x_1+x_2=$_______.

19． Known $\triangle ABC$, then the maximum value of $\sin A+\sin B+\sqrt{6}\sin C$ is_______.

20． Given that positive numbers x and y satisfy $\begin{cases} y^2-4xy+4x\le 0, \\ \frac{y^2}{2}-x^2\ge 1, \end{cases}$

then the minimum value of $\dfrac{x^4+y^3}{3xy+y+1}$ is________.

2019WMTC Advanced Level

Individual Rounds

1	2	3	4	5	6	7
-2	$-2\sqrt{2}$	$\frac{125}{49}$	$\frac{2}{3}$	1	135	32
8	**9**	**10**	**11**	**12**	**13**	**14**
$\frac{4\sqrt{21}}{3}$	$\frac{2019}{4039}$	9	$4\sqrt{3}$	$\frac{25}{4}$	4^{2019}	$\frac{\sqrt{3}-1}{4}$

Relay Rounds

1-B	2-B	3-B
50	$\frac{4}{3}$	$\frac{23}{9}$

Team Round

1	2	3	4	5	6	7	8	9	10
$\frac{2}{5}$	101	$25+120\sqrt{2}$	36	0	3	77	$\frac{\sqrt{5}+3}{2}$	12	21
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
$\frac{673}{2020}$	$\frac{9}{2}$	444	4	2	8	$8\sqrt[3]{4}$	1	$\frac{5}{4}\sqrt{10}$	1