

2018 WMTTC

青年组个人赛第一轮

Advanced Level Individual Round 1

1. Suppose $A = \{x \mid |x| \leq 3, x \in \mathbf{Z}\}$, $B = \{x \mid -x \in A, 2 - x^2 \notin A\}$, then $B = \{\text{_____}\}$.

2. If $3^a = 5$, $5^b = 7$, and $7^c = 9$, then the value of abc is _____.

3. Use $[x]$ to represent the largest integer that is not larger than x . For example, $[2] = 2$, $[-3.1] = -4$, $[\pi] = 3$, then

$$[\sqrt{1}] + [\sqrt{2}] + [\sqrt{3}] + \cdots + [\sqrt{150}] = \text{_____}.$$

4. In $\triangle ABC$, if $\tan A + \tan C + \sqrt{3} = \sqrt{3} \tan A \tan C$, then $\angle B = \text{_____}^\circ$.

5. Suppose $y = f(x)$ is an even function and $f(x) = -f(x+4) + 1$ for any $x \in \mathbf{R}$, then the value of $f(2018)$ is _____.

6. Give a sequence $\{a_n\}$ such that $a_{n+1} = \frac{a_n}{2a_n + 1}$, and $a_{2018} = 1$, then the value of a_{2000} is _____.

7. Fig. 1 is a 5×4 grid that consists of 20 1×1 squares. Suppose $\vec{m} = \lambda \vec{AC} - 3\vec{AB}$ ($\lambda \in \mathbf{R}$), and $\vec{m} \perp \vec{AB}$, then the value of λ is _____.

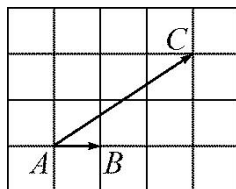


Fig. 1

8. In the Fig.2, suppose $AC=8, BC=6$ and $\angle C=60^\circ$ in $\triangle ABC$, point E on AC , point F on BC . If $S_{\triangle CEF} = \frac{1}{3}S_{\triangle ABC}$, then the minimum value of EF is _____.

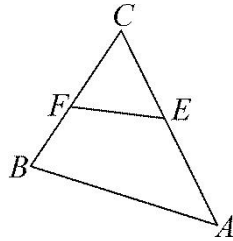
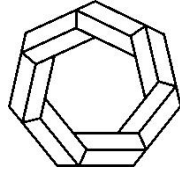


Fig.2



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Advanced Level Individual Round 2

9. If $(x^2 - 2x + \sqrt{3})^4 = a_0 + a_1x + a_2x^2 + \cdots + a_8x^8$, then

$$(a_0 + a_2 + a_4 + a_6 + a_8)^2 - (a_1 + a_3 + a_5 + a_7)^2 = \underline{\hspace{2cm}}.$$

10. In a rectangular box $ABCD - A_1B_1C_1D_1$, $AB = BC = 2$ and $AA_1 = \sqrt{5}$. If the angle of the straight line AD_1 and DB_1 is θ , then $\cos \theta = \underline{\hspace{2cm}}$.

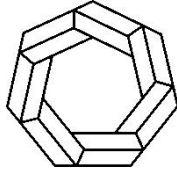
11. Given that $\{a_n\}$ is an arithmetic (equal difference) sequence.

If $a_2 + a_6 + a_7 + a_{12} = 51$ and $a_3 : a_{12} = 1 : 2$, then

$$a_2 + a_4 + a_6 + \cdots + a_{98} + a_{100} = \underline{\hspace{2cm}}.$$

12. Suppose $\theta \in \left(0, \frac{\pi}{2}\right)$, then the minimum value of $\sin^3 \theta - \cos^2 \theta - \sin \theta$

is $\underline{\hspace{2cm}}$.



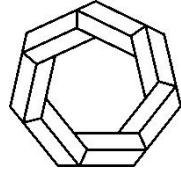
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Advanced Level Individual Round 3

13. Suppose F_1 and F_2 are both foci for the hyperbola $C: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, and that an asymptote of C is perpendicular to line $y = \frac{\sqrt{3}}{3}x + 2$. If a point A on C satisfies $F_1A = \frac{3}{2}F_2A$, then $\cos \angle AF_1F_2 =$ _____.

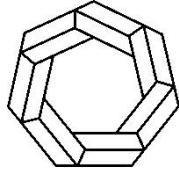
14. Suppose the function $f(x) = \sqrt{a - a^x}$ ($a > 0, a \neq 1$) has $[0, 1]$ as both its domain and range, and the $ABCD$ is a regular tetrahedron with edge length of a , then the radius of the circumscribed sphere of the tetrahedron $ABCD$ is_____.



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Advanced Level Relay Round 1

1-A

If the inequality $\log_a(3+e^{|x|}) \geq \frac{1}{2}$ holds for any real number x , then the value range for a is_____.

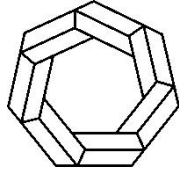


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Advanced Level Relay Round 1

1-B

Let **T** be the number you will receive.

Solve the inequality $2\log_a(x+1) < \log_a 2 + \log_a(3-x)$ using value of a from **T**.



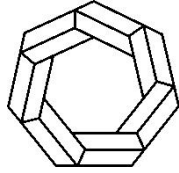
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青年组接力赛第二轮

Advanced Level Relay Round 2

2-A

If $f(x) = \frac{e^x}{e^x + 1}$, then $f(-6) + f(-4) + f(-2) + f(2) + f(4) + f(6) = \underline{\hspace{2cm}}$.

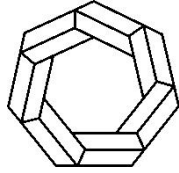


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Advanced Level Relay Round 2

2-B

Let T be the number you will receive.

Suppose $a+b=T$ where a and b are positive real numbers, then the minimum possible value of $\frac{a^2}{a+5} + \frac{b^2}{b+3}$ is _____.

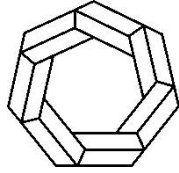


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Advanced Level Relay Round 3

3-A

If a sequence $\{a_n\}$ is defined as $a_1 = 3$, and $a_{n+1} = \frac{1+a_n}{1-a_n}$, then

$$a_{101} = \underline{\hspace{2cm}}.$$

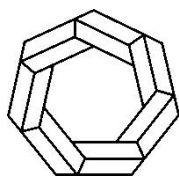


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青年组接力赛第三轮
Advanced Level Relay Round 3

3-B

Let T be the number you will receive.

If $\tan \alpha = T \tan \frac{\pi}{7}$, then $\frac{\cos\left(\alpha - \frac{5\pi}{14}\right)}{\sin\left(\alpha - \frac{\pi}{7}\right)} = \underline{\hspace{2cm}}$.



2018 WMTTC

青年组团体赛

Advanced Level Team Round

1. If $f(x) = \frac{4^x}{4^x + 2}$, then $f\left(\frac{1}{2019}\right) + f\left(\frac{2}{2019}\right) + \cdots + f\left(\frac{2018}{2019}\right) =$ _____.

2. Suppose that real numbers x and y satisfy $x^2 + y^2 \leq 1$, then the maximum value of $x^2 + 4xy - y^2$ is _____.

3. Given two sets $A = \{(x, y) \mid xy \geq 1, x > 0\}$ and $B = \{(x, y) \mid x + 4y \leq a\}$. If $A \cap B = \emptyset$, then the value range for a is _____.

4. Suppose $M = \max\{1 - 2x, 5x - 3, 2x\}$ and $x \in \mathbf{R}$, then the minimal value of M is _____. (Note: $\max\{a, b, c\}$ represents the largest of a, b, c)

5. The root of the equation $\frac{3}{3^x - 1} + \frac{1}{3} = 3^{x-1}$ is $x =$ _____.

6. Suppose the lengths of the three sides that are opposite to the three interior angles A , B , and C of $\triangle ABC$ are a , b , and c , respectively. Suppose $b - c = \frac{1}{4}a$, and $2\sin B = 3\sin C$, then the value of $\cos A$ is _____.

7. The area of the region on the xy -plane over which P ranges, $P \in \{(x, y) \mid ||x| - 1| + |y| = 1, x \in \mathbf{R}, y \in \mathbf{R}\}$, is _____.

8. The number of roots of equation $\sqrt{4 - x^2} - |\ln|x - 1|| = 0$ is _____.

9. Given a sequence $\{a_n\}$ where $a_3 = \frac{1}{15}$ and $a_n - a_{n+1} = 6a_n a_{n+1}$. Then

the formula for $a_n =$ _____.

10. Let G be the center of gravity of $\triangle ABC$, if $GB \perp GC$, and $BC = \sqrt{2}$, then the maximum area of $\triangle ABC$ is _____.

11. In $\triangle ABC$, let O be the midpoint of BC . If

$$AB = 10, AC = 7, \overrightarrow{AM} = \frac{3}{2}\overrightarrow{MB}, \overrightarrow{AN} = \frac{4}{3}\overrightarrow{NC}, \text{ and } \overrightarrow{MO} \cdot \overrightarrow{ON} = 0, \text{ then}$$

$$\cos A = \text{_____}.$$

12. There are four points A, B, C , and D on the sphere with radius 4, and they satisfy $\angle BAC = 90^\circ$, $\angle CAD = 90^\circ$, and $\angle DAB = 90^\circ$. Let S represent the area of the triangle, then the maximum possible value of $S_{\triangle ABC} + S_{\triangle ACD} + S_{\triangle ADB}$ is _____.

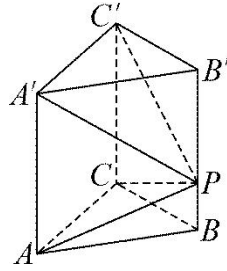
13. If real numbers x and y satisfy
$$\begin{cases} x + y - 7 \leq 0 \\ x - 3y + 1 \leq 0 \\ 3x - y - 5 \geq 0 \end{cases}, \text{ and}$$

$$z = \frac{(y-x)(y+x)}{xy}, \text{ then the value range for } z \text{ is } \text{_____}.$$

14. It is known that the trilateral lengths a, b, c of $\triangle ABC$ satisfy bc, ca, ab in an equal difference sequence. If $a + c = 4$, when the area of $\triangle ABC$ reaches its maximum, the perimeter of $\triangle ABC$ is _____.

15. Suppose $m^2 + n^2 = 20$ and $x^2 + y^2 = 18$ where m, n, x and y are real numbers, then the maximum possible value of $mx - ny$ is _____.

16. The volume of the triangular prism $ABC - A'B'C'$ is 1, and $P \in B'B$, then the volume of the pyramid $P - ACC'A'$ is _____.



17. The sum of all real roots of the equation $x^6 - x^3 + 1 = (x^6 + x^3 + 1)(x^2 + 2x + 4)$ is _____.

18. Suppose $AB = BC = 5$, and $AC = 8$ in $\triangle ABC$, and O is the incenter of $\triangle ABC$. If $\overrightarrow{AO} = m\overrightarrow{AB} + n\overrightarrow{AC}$ ($m, n \in \mathbf{R}$), then the value of $\frac{m}{n}$ is _____.

19. Let S_n be the sum of the first n terms of arithmetic sequences $\{a_n\}$. Suppose $S_m = -2, S_{m+1} = 0$ and $S_{m+2} = 3$ for any integer m , then $a_{2018} =$ _____.

20. Suppose the function $f(x) = x^2 + 2x - 1$ has $[a, b]$ as both its domain and range, then the minimum value of $a + b$ is _____.

2018WMTTC Advanced Level

Individual Rounds

1	2	3	4	5	6	7
-3,3	2	1162	60	$\frac{1}{2}$	$-\frac{1}{35}$	1
8	9	10	11	12	13	14
4	144	$\frac{\sqrt{13}}{39}$	2850	$-\frac{32}{27}$	$\frac{\sqrt{3}}{4}$	$\frac{\sqrt{6}}{2}$

Relay Rounds

1-B	2-B	3-B
(-1,1)	$\frac{9}{11}$	2

Team Round

1	2	3	4	5	6	7	8	9	10
1009	$\sqrt{5}$	$(-\infty, 4)$	$\frac{1}{2}$	$\log_3 4$	$-\frac{1}{4}$	4	4	$\frac{1}{6n-3}$	$\frac{3}{2}$
11	12	13	14	15	16	17	18	19	20
$\frac{3}{8}$	32	$\left[-\frac{21}{10}, \frac{7}{12}\right]$	6	$6\sqrt{10}$	$\frac{2}{3}$	-1	$\frac{8}{5}$	2015	-3