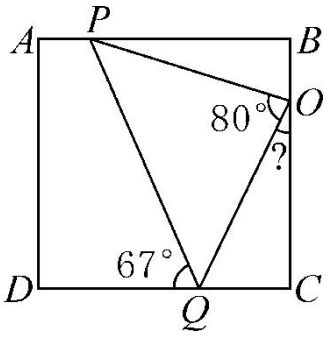
2017 WMTC

少年组个人赛第一轮

Intermediate Level Individual Round 1

1.If $x=\sqrt{3}, y=\sqrt{2}$，then in $(xy)^2$，$(x-y)^{-2}$ and $\frac{x}{x-y}$, which is the biggest value among them?

2. $ABCD$ is a square, and point O, P, and Q are on AB,BC,CD respectively, if OP=OQ,$\angle POQ$=80°,$\angle PQD$=67°,find $\angle QOC$.

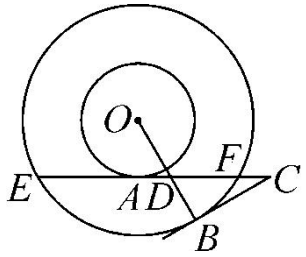


3. If the integer part of $\sqrt{63}$ is m, the decimal part is n, the integer part of $\sqrt{28}$ is p, the decimal part is q. Find the value of $mp-nq$.

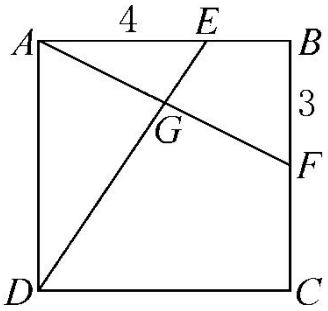
4.Find the value of $\left(\sqrt{5}+1\right)^{2017}-2\left(\sqrt{5}+1\right)^{2016}-4\left(\sqrt{5}+1\right)^{2015}+2014$.

5.If x， y are positive integers, and $2x+y=20$. Find the maximum value of $\frac{y}{x}$.

6. Point O is the center of two concentric circles, their radius are r and $2r$. The chord EF and small circle are tangent to the point A, the point C is on the extension line of EF, CB and the great circle are tangent to point B, OB and EF intersect at point D. If $\angle DCB=30°$, find $\frac{OD}{ED}$.



7.$ABCD$ is a square, point E, F on AB, BC respectively, AF and DE intersect at point G, if AB=6, AE=4, and BF=3, find the area of $CDGF$.



8. Take 9 numbers from 0,1,2,3,4,5,6,7,8,9, and use them to make a two digit number, a three digit number, and a four digit number, if the sum of them is 2017, find the number that didn't be taken out.

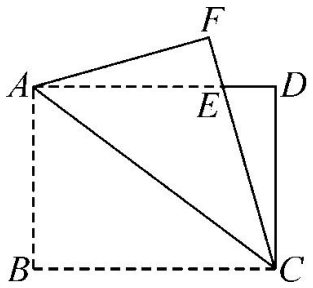
2017 WMTC

少年组个人赛第二轮

Intermediate Level Individual Round 2

9.Point M is the midpoint of AC, the semicircle M and BC are tangent to the point D, the semicircle N and the semicircle M are tangent to the point E, and the point B is on the semicircle N. If AB=4,CB=8,then find the area of the shadow part. ($\pi = 3$)

10. $ABCD$ is a rectangle, fold it along the line AC, and point B falls at the point F, CF and AD intersect at point E. Find the area of $\triangle ACE$.



11. The unknown number of the inequality $6x^2 - ax - a^2 < 0$ is x, and this inequality has only 7 integer solutions. Find positive integer a.

12.If a, b, c are real numbers, and $abc \neq 0$, $\begin{cases} 2a-2b+9c=9, \\ a-2b+6c=5, \end{cases}$ find the value of $\frac{a+3c}{a+4b-3c}$.

2017 WMTC

少年组个人赛第三轮

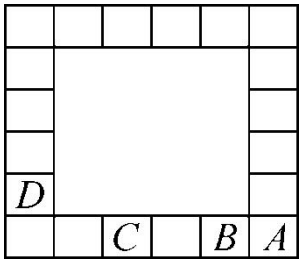
Intermediate Level Individual Round 3

13. If a, b, and c are positive integers, if

$$abc + ab + bc + ac + a + b + c = 3144,$$

find the value of $a + b + c$.

14.A child starting from A, for the first time he walks 1 lattice arrive at B, for the second time he walks 2 lattices arrive at C, and for the third time he walks 3 lattices arrive at D, ⋯. How many times did he go when he went back to A for the third time after finish that time?



2017 WMTC

少年组接力赛第一轮

Intermediate Level Relay Round 1

1-A

If x and y are positive integers, and $2^x + 3^y = 59$, find the value of xy.

2017 WMTC

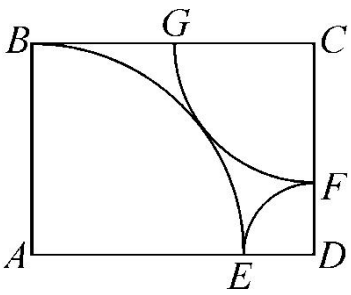
少年组接力赛第一轮

Intermediate Level Relay Round 1

1-B

Let T be the number you will receive.

Known $ABCD$ is a rectangle. $AB = T$, $\overset{\frown}{BE}$ is $\frac{1}{4}$ of circle A, $\overset{\frown}{EF}$ is $\frac{1}{4}$ of circle D, $\overset{\frown}{GF}$ is $\frac{1}{4}$ of circle C, tangency $\overset{\frown}{BE}$ and $\overset{\frown}{EF}$, find the length of BC.



2017 WMTC

少年组接力赛第二轮

Intermediate Level Relay Round 2

2-A

Known A, B, and C are real numbers, if $\frac{2x^2+1}{x^3-1}=\frac{A}{x-1}+\frac{Bx+C}{x^2+x+1}$, find the value of $(A-3B+C)^2$.

2017 WMTC

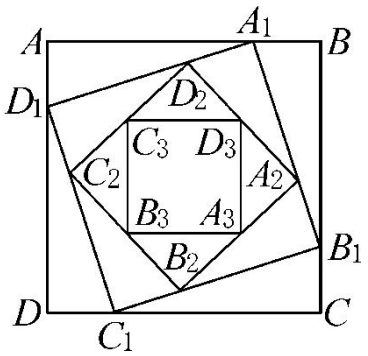
少年组接力赛第二轮

Intermediate Level Relay Round 2

2-B

Let T be the number you will receive.

Known $ABCD$, $A_1B_1C_1D_1$, $A_2B_2C_2D_2$, and $A_3B_3C_3D_3$ are all squares, and A_1，B_1，C_1，D_1 are all four equal points, A_2，B_2，C_2，D_2 are all three equal points, A_3，B_3，C_3，D_3 are all midpoints. If $AB = T$, find the area of $A_3B_3C_3D_3$.



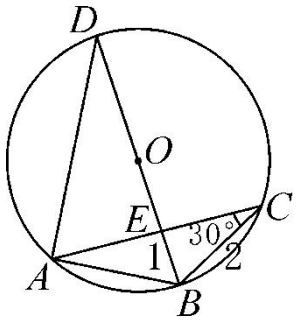
2017 WMTC

少年组接力赛第三轮

Intermediate Level Relay Round 3

3-A

Known BD is the diameter of circle O, points A，C are on circle O. If BC=2，BE=1，$\angle ACB$=30°, find the length of AD.



2017 WMTC

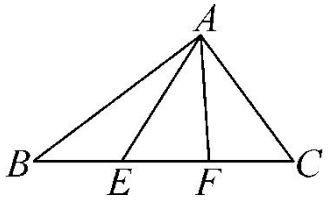
少年组接力赛第三轮

Intermediate Level Relay Round 3

3-B

Let T be the number you will receive.

Known triangle BAC, and $\angle BAC = 90^\circ$, $BC=T$, E and F on BC, and they are three equal points. Find the value of $AE^2 + AF^2$.



2017 WMTC

少年组团体赛

Intermediate Level Team Round

1. *ABCD* is a quadrilateral and $A>B>C>D$, if α is the minimum of *A- B*,*B- C*,*C- D*, and 115°- *A*. Find the maximum of α.

2.If *a*， *b* are real numbers, and $|2a-1|+\sqrt{(a-2)b^2}+\sqrt{b+4}+1=2a$, find the value of a^2b+ab^2.

3. If $xy\neq 1$, $19x^2+2017x+5=0$, and $5y^2+2017y+19=0$. Find the value of $\dfrac{x}{xy+1}$.

4.There are 9 unit squares in the Fig.1, and take 3 out of them random, find the probability that it is an axial symmetric figure.

（Example: Fig.2 is an axial symmetric figure; Fig.3 isn't an axial symmetric figure）

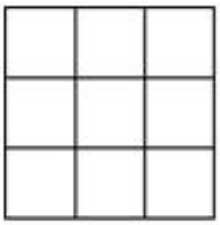


Fig.1

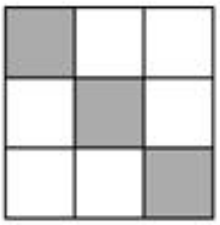


Fig.2

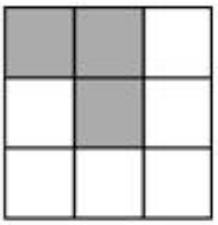


Fig.3

5.There are 15 circles in Fig.1, and any 3 circles like Fig.2 can make $x=\sqrt{2}y+z$ .Find *A*.

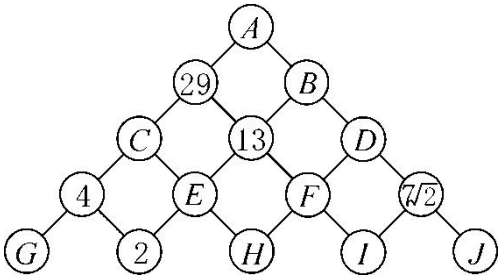


Fig.1

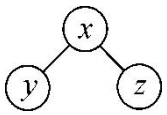
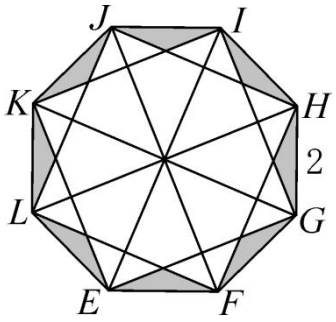
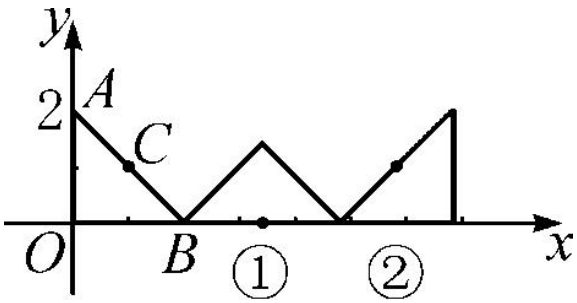


Fig.2

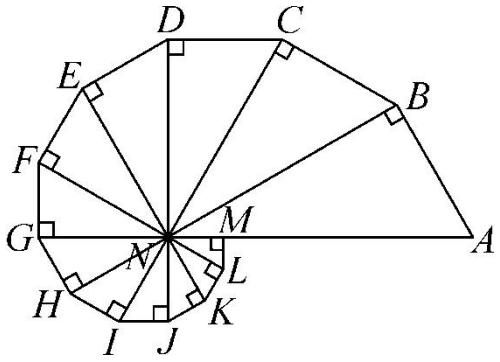
6.*EFGHIJKL* is a regular octagon, if *EF*=2, find the area of the shadow part.



7.Known △*AOB*, *AO*=*OB*, ∠*O*=90°, point *C* is midpoint of *AB*. Rotate it along the *x* axis to the right without sliding, first times to the position ①, second to the position ②,…, and when it rotates 2017 times, find the coordinates of the point *C*.



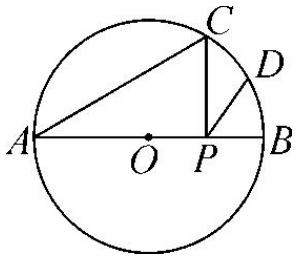
8.In the graph, there are 12 similar right triangles, and point *N* is their common vertex. If *ML*= $\left(\frac{\sqrt{3}}{2}\right)^{11}$, find the length of *AB*.



9. *m* and *n* are positive integers, $5^m \times 4^n$ is a 28-digit number. If *m*>25, find the value of *m*+*n*.

10.The length of the three sides of the right trapezoid is 6, 6, 8. Find the minimum area of the trapezoid.

11.In the graph , *AB* is diameter of $\odot O$, point *C* on $\odot O, \angle CAB = 30°$, point *D* is midpoint of $\overset{\frown}{CB}$, point *P* on *AB*, if minimum of $PC + PD$ is 1，find the radius of $\odot O$.



12.If *n* is a positive integer, and $(n+1)^3 - n^3 \le 2017$, find the maximum of *n*.

13.Known *A*， *B*， and *C* are one digits numbers, and $ABC \neq 0$, if $\overline{ABAA} = \overline{BA} \times \overline{CAB}$, find $\overline{CAB}$.

14. If the real numbers x, y, z satisfy

$$4\left(\sqrt{x-1}+\sqrt{y-3}+\sqrt{z-8}\right)=x+y+z,$$

find the value of $x+2y-3z$.

15. Known a， b are positive integers, in the enclosed area enclosed by straight line $y=a-x$ and hyperbola $y=\frac{b}{x}$, there are 123 points that abscissa and ordinate are positive integer, find the minimum value of $a+b$.

16.Solve equation $\sqrt[3]{190-x}+\sqrt[3]{27+x}=7$ ($x>0$).

17. Known x, y satisfy $(\sqrt{x}-\sqrt{x-2018})(\sqrt{y}-\sqrt{y-2018})=2018$, find the value of $4x^2-4y^2+5x-4y-2017$.

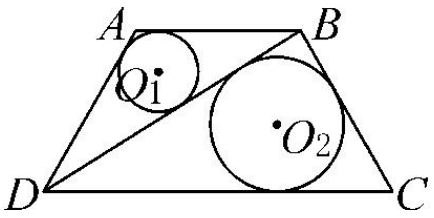
18. Known a, b are prime numbers satisfy

$$8(2a+3b)^2=127(21a+349b),$$

find the value of $a+b$.

19.Known a is positive integer, $\sqrt{68+a}$ and $\sqrt{68-a}$ are rational numbers, find a.

20.Known $ABCD$ is isosceles trapezoid, $AB\ //\ CD$， $\angle C=60°$， $\odot O_1$ and $\odot O_2$ are inscribed circles of ΔABD and ΔBCD, $r_1=3$, $r_2=5$, find the height of $ABCD$.



2017WMTC Intermediate Level

Individual Rounds

1	2	3	4	5	6	7
$5+2\sqrt{6}$	27°	$29\sqrt{7}-42$	2014	18	$\frac{1}{2}$	18
8	**9**	**10**	**11**	**12**	**13**	**14**
8	$\frac{22}{3}$	$\frac{75}{16}$	9	2	56	39

Relay Rounds

1-B	2-B	3-B
20	$\frac{25}{9}$	$\frac{20}{3}$

Team Round

1	2	3	4	5	6	7	8	9	10
10°	16	$-\frac{5}{2017}$	$\frac{3}{7}$	$54\sqrt{2}$	$8\sqrt{2}$ -8	$(1345\sqrt{2}+2690,0)$	1	41 or 42	$36-6\sqrt{7}$
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
$\frac{\sqrt{2}}{2}$	25	153	-17	27	189	1	288	32	12