2017 WMTC

青年组个人赛第一轮

Advanced Level Individual Round 1

1.Known n is a positive integer number, and $\sqrt{2}$ is between $\frac{n+4}{n+2}$ and $\frac{n+3}{n+1}$. Find n.

2.Known function $f(x)=|x+1|+|x-1|+\frac{1}{2}\left(\mathrm{e}^{x}+\mathrm{e}^{-x}\right)\left(x\in\mathbf{R}\right)$, find minimum of $f(x)$.

3.The vertex A,B of a square are also the vertex of an ellipse M, and the vertex C,D of this square are the focus of M. Find eccentricity of M.

4.Known series $\{a_n\}$ satisfy $a_{n+1}=\begin{cases}2a_n, & 0\le a_n<\frac{1}{2};\\ 2a_n-1, & \frac{1}{2}\le a_n<1.\end{cases}$ If $a_1=\frac{2}{7}$,then a_{2017}=?

5.The area of the right triangle ABC is $\frac{1}{2}$. Find the minimum value of the triangle's perimeter.

6.Suppose that $[x]$ is the largest integer not greater than x .Find the value of $[\lg 1]+[\lg 2]+[\lg 3]+\cdots+[\lg 2017]$.

7.Find the value of $\tan^2 25^\circ + 2\tan 25^\circ \tan 40^\circ$.

8.Known $ABCD - A_1B_1C_1D_1$ is a cuboid, and $AB=BC=2$, $AA_1=1$, α is the angle formed by AC_1 and BB_1. Find the value of $\cos\alpha$.

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Advanced Level Individual Round 2

9.Suppose x_n($n \in \mathbf{N}^*$) is a positive integer, if $x_{n+2}=x_n+x_{n+1}$, $x_6=61$, x_1 is a prime number. Find the maximum value of x_1.

10.The point M on the ellipse $\frac{x^2}{8}+\frac{y^2}{4}=1$,and the point F is right focus of this ellipse, point $P(2,1)$. Find the minimum value of $\sqrt{2}\,|MF|+|MP|$.

11.Know $ABCD-A_1B_1C_1D_1$ is a cuboid, if $AB\cdot AD=12$, $AB\cdot AA_1=36$, $AD\cdot AA_1=48$.If points A,B,C,D,A_1,B_1,C_1,D_1 are on the sphere O,find the surface area of sphere O.

12.If x and y are integer, and $x^3+6x^2+5x=y^3-y+2$ .Find the number of (x,y).

2017 WMTC

青年组个人赛第三轮

Advanced Level Individual Round 3

13.Known the point P is on the unit circle O, $A_1A_2 \cdots A_{2017}$ is inscribed regular 2017 polygon of circle O. Find the value of $PA_1^2 + PA_2^2 + \cdots + PA_{2017}^2$.

14.Known x_1=2, and $x_{n+1} = \dfrac{x_n^{\,2}}{x_n + 1}$ (n=1,2,3, $\cdots$). Find integer part of $\sum_{n=1}^{2017} \dfrac{x_n}{x_n + 1}$.

2017 WMTC

青年组接力赛第一轮

Advanced Level Relay Round 1

1-A

S_n is the sum of first n number of arithmetic progression $\{a_n\}$, if $a_1 = 1$, the difference between any two adjacent number is 2, and $S_{m+2} - S_m = 36$, find m.

2017 WMTC

青年组接力赛第一轮

Advanced Level Relay Round 1

1-B

Let ***T*** be the number you will receive.

Known triangle pyramid *P-ABC,PA=PB=PC,AB=BC=CA,*$\angle BPA = \angle APC = \angle CPB = 90°$,area of $\triangle APB$ is ***T***. Find area of circumscribed sphere of *P-ABC*.

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青年组接力赛第二轮

Advanced Level Relay Round 2

2-A

Suppose A is the region enclose of equation $|x-2|+|x-1|+|y|=3$. Find area of A.

2017 WMTC

青年组接力赛第二轮

Advanced Level Relay Round 2

2-B

Let ***T*** be the number you will receive.

The radius of the sphere O is ***T***, the point P,A,B,C are on the sphere O, and PA,PB,PC are perpendicular to each other, and the height of the triangle pyramid P-ABC is h. Find maximum of the h.

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青年组接力赛第三轮

Advanced Level Relay Round 3

3-A

Suppose $ab>0$, $a+b=ab$, Find minimum value of $\frac{a}{1+a}+\frac{b}{1+b}$.

2017 WMTC

青年组接力赛第三轮

Advanced Level Relay Round 3

3-B

Let ***T*** be the number you will receive.

Point F is the focus of parabola $x^2 = 2py(p > 0)$, the line whose angle of inclination is 60° intersect parabola at point A,B, if area of $\triangle OAB$ is ***T***. Find the value of p.(O is the origin of coordinate system)

2017 WMTC

青年组团体赛

Advanced Level Team Round

1.Suppose $a_n = 2n+1$ (n=1,2,3, $\cdots$). Calculate:

$$a_1a_2 - a_2a_3 + a_3a_4 - a_4a_5 + \cdots + a_{17}a_{18}.$$

2.Known function $f(x) = \dfrac{2^{x+2}+2}{2^x+1}$,when -4≤x≤4, M and m is the maximum and minimum of $f(x)$， respectively. Find $M + m$.

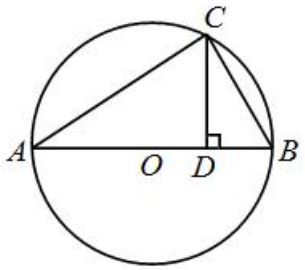
3. $ABCD - A_1B_1C_1D_1$ is a cuboid, point E is on the edge B_1D_1,point F is on the AE, and AF=2FE. If AB=4, AD=2, AA_1=3. Find the volume of tetrahedron $BDEF$.

4.Known real number a,b,c, and $\begin{cases} a(4-b)=4, \\ b(4-c)=4, \\ c(4-a)=4. \end{cases}$ find the value of a+b+c.

5.Known function $f(x) = ax^2 + bx + c$,when $d \le x \le d+2$,

$e \le f(x) \le e+1$, a,b,c,d,e are real numbers. Find the maximum value of a.

6.Known AB is diameter of circle O, point C is on the circle O, $CD \perp AB$. Find the possible that AD,BD,CD can make an acute triangle.



7.Suppose x is a real number, continuous function $f(x)$ satisfy:

(1) $f\left(x^3\right)=\left(f(x)\right)^3$;

(2)If $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$.

Find the value of $\left[f(-1)+f(1)\right]^2 - f(0)$.

8.Known a,b,c,d are distrinct positive integers. Find the minimum value of $\dfrac{abcd}{a+b+c+d}$.

9.Solve the equation: $\sqrt{x+\dfrac{x^2}{x+1}}+\dfrac{x}{\sqrt{x+1}}=x+1$.

10.Known arithmetic sequence $\{a_n\}$ and $\{b_n\}$, S_n and T_n is the sum from a_1 to a_n,b_1 to b_n, respectively, and $\dfrac{S_n}{T_n}=\dfrac{2n+4}{3n-1}$. Find the value of

$\frac{a_6}{b_5}$.

11.Let $ab>0$, and $a+2b=1$. Find the maximum value of $a+\sqrt{ab}$.

12.Known $A(-3,0)$,$B(-1,-2)$, point C is on ellipse $\frac{x^2}{4}+\frac{y^2}{3}=1$.Find the minimum value of the area of $\triangle ABC$.

13.Suppose positive integer a_n and 190 are coprime, and $a_1<a_2<a_3<\cdots<a_n$. Find a_{2017}.

14.Point A,B,C,D are not in the same plane, and $AB=1$, $BC=2$, $CD=3$, $AC\perp BD$. Find the length of AD.

15.Suppose function $f(x)=x^2-2ax+a^2-1$,if there is only one x can make $f(f(x))\leqslant 0$,find the range of a.

16.Known the line $l:2x-y-6=0$ and the parabola $C:y^2=2px$ intersect at A, B, point F is the focus of C, if $\overrightarrow{FA}\cdot\overrightarrow{FB}=0$, find the value of p.

17.Known the point P is on the image of the function $y=\mathrm{e}^{x}$,the image is tangent line l at the point P , and the line l, line x=1,x=2 and x axis form a trapezoid. Find the maximum of area of the trapezoid.

18.The football is made of x pieces of the same regular pentagon and y pieces of the same regular hexagon, and x+y=32. Find the value of x.

19.Known a,b,c are all positive numbers, and $\sqrt{\frac{a^2}{a^2+b^2}}+c\sqrt{\frac{b}{a+b}}\le\frac{3\sqrt{2}}{2}$.Find the maximum value of c.

20.Suppose function $f(x)=\sum_{k=1}^{2017}\left[\frac{x}{k!}\right]$,$[x]$ is the largest integer less than x. If the equation $f(x)=n$ ($1\le n\le 2017$ and n is an even number) has solution. Find out how many n can make it.

$$\left(\sum_{k=1}^{2017}\left[\frac{x}{k!}\right]=\left[\frac{x}{1!}\right]+\left[\frac{x}{2!}\right]+\cdots+\left[\frac{x}{2017!}\right]\right)$$

2017WMTC Advanced Level

Individual Rounds

1	2	3	4	5	6	7
3	3	$\frac{\sqrt{2}}{2}$	$\frac{2}{7}$	$2+\sqrt{2}$	4944	1
8	**9**	**10**	**11**	**12**	**13**	**14**
$\frac{1}{3}$	17	2	507	0	4034	1

Relay Rounds

1-B	2-B	3-B
144	$\frac{16}{3}$	$\frac{2\sqrt{3}}{3}$

Team Round

1	2	3	4	5	6	7	8	9	10
687	6	2	6	1	$\sqrt{5}-2$	0	$\frac{12}{5}$	$\frac{1+\sqrt{5}}{2}$	1
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
$\frac{2+\sqrt{6}}{4}$	$3-\sqrt{7}$	5321	$\sqrt{6}$	$\{-2\}$	1	$e^{\frac{3}{2}}$	12	2	587