2015WMTC Junior Level

Team Round Problems

1. Find the sum of the digits of the quotient $\underbrace{111\cdots11}_{2015}\ \underbrace{222\cdots22}_{2015}\ \underbrace{333\cdots33}_{2015} \div \underbrace{333\cdots33}_{2015}$.

2. Use five small identical rectangles to form a large rectangle with perimeter of 160 as shown in figure 1. What is the area of the smallest square that can be formed using this kind of large rectangles?

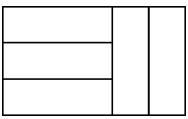

Fig.1

3. Given two single decimal numbers $\square$ and $\triangle$ where $\square$ and $\triangle$ represent digits from 1 to 9. If we round the product ($\square \times \triangle$) into a single decimal number, this product becomes 22.3. Find the actual product before rounding.

4. For each binary number (base 2) with 2015 as the sum of its digits, rewrite this number in base 8 and compute the sum of its digits. Among all such possible base 8 numbers and their sums of digits, what is the minimum sum?

5. Given a stack of 5 cards each is written with a different number from 0, 1, 2, 4, or 5. Choose 1 or 2 or 3 or 4 or 5 cards from this stack of cards and form a 1-digit, 2-digit, 4-digit, or 5-digit number. Discard any number formed in this manner that contains 2, 0, 1, and 5 as digits and duplicate numbers. If the remaining numbers are ordered from large to small, what is the 123rd number?

6. Consider the 4×4 grid square in figure 2. Any intersections of horizontal and vertical lines are called "grid points" which include the points on the perimeter. How many rectangles (including squares) of area 4 can be formed using these "grid points" as vertices?

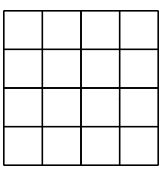

Fig.2

7. The figure 3 shows a 60×100 (60 rows and 100 columns) grid figure. Each intersection point of vertical and horizontal lines is called a "grid point." Let A and B be the diagonally opposite corner grid points. If A and B are connected with a straight line, how many grid points (counting A and B) would this straight passes through?

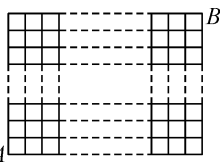

Fig.3

8. If the seven digit number $\overline{20xy15z}$ is divisible by 792, find $x + y + z$.

9. Suppose the sum of the dividend, divisor, quotient, and remainder of one particular division is 181 and its divisor is the same as the product of its quotient and remainder. Find its dividend.

10. As shown in figure 4, the circle is the circle with the largest area inside square $ABCD$ and rectangle $EFGH$ is the square with the largest area inside that circle. If the difference of the areas of these two squares is 16 and the difference of their perimeters is 8, find the area of the shaded region. (use $\pi = 3$)

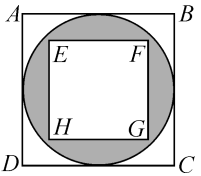

Fig.4

11. Consider the figure 5. Supposea rectangle is cutting into two identical pieces in the shape of "⌐" and that these two pieces of "⌐" can be composed together to form a square. If the sum of the perimeters of these two piecesis 2016 more than the perimeter of the original rectangle, find the side length of the composed square.

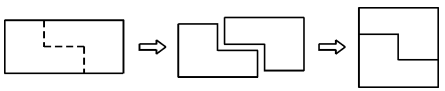
Fig.5

12. Given that both $\overline{MA}$ and $\overline{TH}$ are 2-digit numbers and different letters represent different digits. If $\overline{MA}+\overline{TH}=89$, find the units digit of $(M+A+T+H)^{2016}$. (Note: n^{2016} means the product of 2016 n's)

13. Observe the pattern of the following table. If the last row has only one number which we call it x, find the number of factors of x.

1	2	3	4	5	…	95	96	97	98	99
	4	6	8	10	…	190	192	194	196	
		12	16	20	…	380	384	388		
			…	…	…	…	…			
				…	…	…				
					x					

14. A store is selling pears and apples. The number of apples in this store is 10 less than 3 times the number of pears it has during the first day. If this store can sell 30 pears and 70 apples every day, 8 days later, the number of apples in the store is 30 more than 5 times the number of pears it has. How many apples does it have originally?

15. How many different values does $\left[\frac{n}{2}\right]+\left[\frac{n}{3}\right]+\left[\frac{n}{5}\right]$ take if n is a natural number that takes on values from 1 to 2015? (Note: $[x]$ represents the largest integer not greater than x).

16. As shown in figure 6, square $ABCD$ and square $EFGH$ overlapped in the shaded region. Suppose $AM=MB$, $CN=2NB$, and $EM=MN=NH$. If this overlapped region has an area of 1, find the difference of areas of the two non-overlapped regions of these squares.

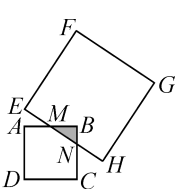

Fig.6

17. Each circle in figure 7 is painted with one color. How many different colors are needed to paint these circles so that any two circles that are connected by a straight line will have different colors?

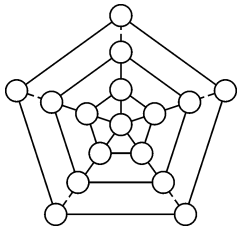

Fig.7

18. Five people A, B, C, D, and E are involved in a gift exchange program under the following two conditions: (1) If a person receives a gift, he must return a gift to the original giver, (2) Gift exchange between two people can only happen 0 or 1 time. If A receives 4 gifts, B receives 2 gifts, D receives 1 gift, and E receives 3 gifts, how many gifts C would receive based on the above two conditions.

19. Suppose A, B, and C start from the same position running on a track at the same time trying to catch up to D who is already running at the time. Given that the running speeds of A, B, and C are 100 meters per minute, 120 meters per minute, and 90 meters per minute, respectively. If it takes A 50 minutes and B 30 minutes to catch up to D, how long does it take for C to catch up to D?

20. The figure 8 is a 7×7 grid figure and each grid point can be identified by a pair of numbers which we called them "coordinates." For examples, $A(0, 0)$, $B(7, 7)$, $C(3, 2)$, and $D(5, 4)$. Suppose an ant starts from A and crawling either up or to the right passing through points C and D going toward point B. If there is a "breakpoint" (a grid point where the ant cannot pass) E somewhere in this grid figure and if there is a total of 300 different paths for the ant to go from A to B, find the coordinates of point E.

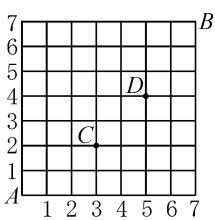

Fig.8

Team Round Answers

1. 6047.
2. 22500.
3. 22.26.
4. 2015.
5. 120.
6. 25.
7. 21.
8. 10.
9. 150.
10. $\frac{39}{4}$.
11. 864.
12. 1.
13. 153.
14. 890.
15. 1479.
16. 27.
17. 4.
18. 2.
19. 75.
20. (4,3).

Relay Round Problems

1A. Define $a\&b = a \times a - b \times b$. Find $100\&99$.

1B. Let $T = \text{TR}$ (The number you will Receive).

Given a piece of rectangular paper of length T and width 120. Cut this piece of paper into two parts along the dotted line as shown in figure 1. Find the sum of the perimeters of both parts.

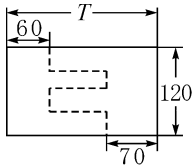

Fig.1

Second Round

2A. If both 2-digit numbers $\overline{ab}$ and $\overline{ba}$ $(a \neq b)$ are prime numbers, we would call them "WMTC numbers". How many possible 2-digit WMTC numbers are there?

2B. Let $T = \text{TR}$. As shown in figure 2, cut out two circles from a piece of rectangular piece of paper so that the area of the large circle is 9 times as large as the area of the small circle. If the diameter of the small circle is T, find the area of the shaded (rectangular) region.

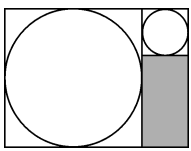

Fig.2

Third Round

3A. If the remainder is 3 when a is divided by 11 and the remainder is 5 when b is divided by 11, find the remainder when $4a + 3b$ is divided by 11.

3B. Let $T = \text{TR}$. A shop needs to produce a number of pairs of gloves. Suppose A can finish this job by himself in 30 hours, B can finish this job by himself in 40 hours, and C can finish this job by herself in 60 hours. If A, B, and C started to work at 8 o'clock and A completed Tmore pairs of gloves than B at 10 o'clock, how many more pairs of gloves did A complete more than C at 12 o'clock?

Relay Round Answers

1A. 199.

1B. 1292.

2A. 8.

2B. 128.

3A. 5.

3B. 20.

Individual Round Problems

First Round

1. Find the sum $19+198+1987$.

2. As shown in figure 1, 11 identical size square cards are placed on a desk to form a figure of "2". Find the number of rectangles in this figure with an area of 2.

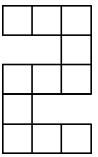

Fig.1

3. How many consecutive zeros are at the end of the product

$$25\times26\times27\times28\times29\times30.$$

4. Suppose there 5 natural numbers with an average of 40. If 10 is one of these numbers and it is changed into another number making the new average to be 45, find this new changed number.

5. Among all 2-digit numbers that are formed by choosing any two different numbers from 1, 2, 3, 4, and 5, how many are multiples of 3?

6. As shown in figure 2, $ABCD$ is a rectangle and $AEFG$ is a square of side length of 10. If $EB=8$ and $GD=4$, find the area of the shaded portion.

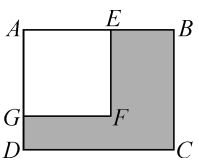

Fig.2

7. Suppose A can input 70 words per minute and B can input 74 words per minute. How long would it take for them to input a total of 14400 words?

8. A has four of each 10 and 5 gram weights. If she takes out one or more weights from these 8 weights, how many different total weights can she have?

9. The figure 3 is composed of 5 rectangles and 4 identical sized fans with the indicated dimensions. Find the total area of shaded regions. (use 3.14 for π)

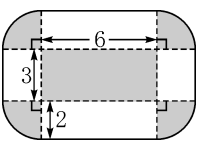

Fig.3

10. How many factors are shared by both numbers 48 and 168?

11. Three years ago, father's age was 15 times son's age. Two years from today, father's age will be 5 times son's age. What is son's age today?

12. The figure 4 shows a solid that is composed by 10 cubes of edge length of 1. Find the surface area of this solid (include the bottom surface area).

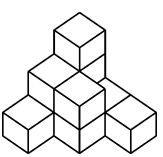

Fig.4

13. Let a and b be natural numbers such that $a>b$. If the Greatest Common Factor(GCF) of a and b is 42 and their Least Common Multiple (LCM) is 252, find the value of a when $(a-b)$ is smallest.

14. There are 20 people participating in an archery competition. Eighteen of them hit the target in the first round, fifteen hit the target in the second round, and only 10 people hit the target in the third round. What is the minimum possible number of people who hit the target in all three rounds?

Second Round

15. As shown in figure 5, $BM=\frac{1}{5}AB$, $BD=\frac{1}{3}BC$, $S_{\triangle MCD}=4$, $AD=6$, and $AD\perp BC$. Find CD.

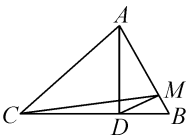

Fig.5

16. A school is holding a table tennis tournament. In the beginning, $\frac{3}{11}$ of the students in the school signed up to compete. Later, another 136 students signed up. At that time, the total number of students who signed up is $\frac{7}{4}$ times the number of students who did not sign up. How many students are in the school?

17. Suppose the letters W, M, T, and C represent 4 different digits and $\overline{WW}\times\overline{MM}+\overline{TW}+C=2015$. Find the value for $W+M+T+C$.

18. As shown in figure 6, let the area of square $ABCD$ be 124 and the area of quadrilateral $EFGH$ be 16. Find the sum of the areas of the two shaded regions.

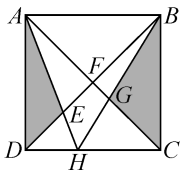

Fig.6

Third Round

19. How many triangles are in figure 7?

20. As shown in figure 8, each of these black and white boxes contains a certain number of balls of same color as the box. Suppose each black box contains no more than 50 black balls and each white box contains a different number of white balls. If each box has at least one ball and the total number of balls in all these boxes is 2015, what is the maximum total number of black balls in all these black boxes?

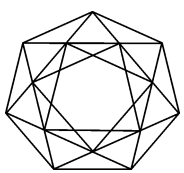
Fig.7


Fig.8

Individual Round Answers

1. 2204.

2. 10.

3. 3.

4. 35.

5. 8.

6. 152.

7. 100.

8. 12.

9. 30. 56.

10. 8.

11. 5.

12. 36.

13. 126.

14. 3.

15. $\frac{20}{3}$.

16. 374.

17. 18.

18. 47.

19. 84.

20. 1550.