2015WMTC Intermediate Level

Team Round Problems

1. Given a rectangle with an area of 910 and both its length and width are integers larger than 20. Find an integer that is closest to the length of its diagonal.

2. Suppose a rectangle has an area of 2016 and both its length and width are integers. Find the perimeter of such rectangle with smallest difference between its length and width.

3. How many possible pairs of non-zero real numbers a and b so that there are exactly two different values among the four numbers $a+b$, $a-b$, $a\times b$, and $a\div b$?

4. Given $a>0$ and $b>0$. If $a+b=3$, find the smallest value for

$$\frac{a^2+4}{a}+\frac{b^2}{b+3}.$$

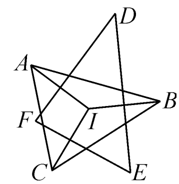


Fig.1

5. Let I be the center of the circumcircle of $\triangle ABC$ and $\triangle DEF$ is formed by using the perpendicular bisectors of IA, IB, and IC as its three sides as shown in figure 1. If $IA=6$ and $S_{\triangle DEF}=21$, find the perimeter of $\triangle DEF$.

6. If $x=\sqrt[3]{16}+\sqrt[3]{12}+\sqrt[3]{9}$, find $x-\frac{1}{x^2}$.

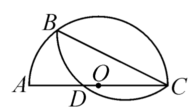


Fig.2

7. Given a semi-circle O. Let BC (not its diameter) be a chord and $\overset{\frown}{BC}$ be its minor arc. As shown in figure 2, use the chord BC as axis of symmetry and fold the minor arc $\overset{\frown}{BC}$ over until it intersects the diameter AC at point D. If $\frac{AD}{AC}=\frac{2}{5}$ and $AC=2015$, find the length of chord BC.

8. Randomly select a positive factor from 6^{2015}. If the probability of this factor happens to be a multiple of 6^{1512} is $\frac{n}{m}$ where m and n are relatively primes, find the value of $m-n$.

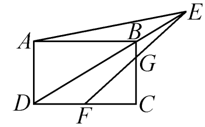


Fig.3

9. As shown in figure 3, point E is on the extension of rectangle $ABCD$'s diagonal DB with $DB=2BE$. Let F be the midpoint of DC and EF intersects BC at G. If the area of $\triangle AEB$ is 100, find the area of $\triangle BEG$.

10. Let S_m be the area of the triangular region that is enclosed by straight lines l_1: $y=mx+2(m-1)$, l_2: $y=(m+1)x+2m$, and the x-axis where $m=1, 2, 3, \cdots$. Find the value of $S_1+S_2+S_3+\cdots+S_{2015}$.

11. An inverse proportion function $y=\frac{k}{x}$ has two points A and B on the First Quadrant. Draw a line segment AD that is perpendicular to the y-axis at D and another segment BC that is perpendicular to x-axis at C as shown in figure 4. If the area of $\triangle OAB$ is $\frac{5}{6}$ and the area of $\triangle OCD$ is $\frac{3}{2}$, find k.

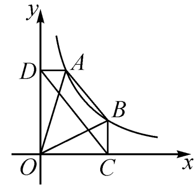


Fig.4

12. Suppose the sum of k consecutive positive integers is 2015. Find the smallest number among these k numbers.

13. Suppose M is a positive integer and both $8M+40$ and $8M-40$ are perfect squares. Find the value of M.

14. Consider the figure 5. Suppose the inscribed circle of $\triangle ABC$ has a radius of 2. Let M and N be points on AB and BC so that they are the intersections of the line that passes through the center and parallel to AC. If $MN=7$ and $AM=4$, find the area of the trapezoid $AMNC$.

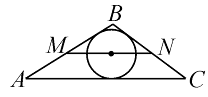


Fig.5

15. Suppose the equation $ax^2+bx+c=0$ has real solutions and its coefficients a, b, and c satisfy the following conditions:

(1) a, b, and c are positive integers;

(2) The 6-digit number $\overline{a2015b}$ is divisible by 12;

(3) c^3+3 is divisible by $c+3$.

Find the maximum value for $a+b+c$.

16. As shown in figure 6, a ray of light enters from point A of a $4\times m$ grid graph. This ray will reflect whenever it hits the sides AB, BC, CD, or AD. However, if would leave the graph when it hits the corner points A, B, C, or D. Suppose this ray of light enters from A and passes through 2016 grid points (including points A and D and each point would only count once) and then leave the graph at point D. Find m.

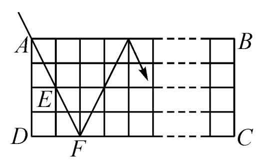


Fig.6

17. If positive integers x and y satisfy the equation $x^3+5x^2y+8xy^2+6y^3=91$, find the value of $x+y$.

18. Suppose a circle of radius 1 that is the inscribed circle of a regular hexagon and also the circumcircle of a square. Let a and b be the edge lengths of the hexagon and square, respectively. If the line $y=-\frac{b}{a}x+\frac{a}{b}$ forms a triangle with x-axis and y-axis and the inscribed circle of this triangle has a radius of r, find the value of $\frac{1}{r}$.

19. Consider the figure 7. Given three points $A(-3, 0)$, $B(\sqrt{3}, 0)$, and $C(0, -3)$. How many possible points $E(x, y)$ (where $0 < y < 4$) that will make $\triangle ABE$ similar to $\triangle ABC$?

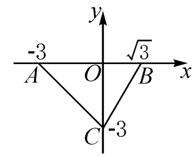


Fig.7

20. As shown in figure 8, point B is the midpoint of arc $\overset{\frown}{AC}$, point E is on the chord AC, and F is on arc $\overset{\frown}{AC}$. If arc $\overset{\frown}{AC} = 120°$, $\angle B = \angle FEC = 90°$, and the radius of the circle for the arc $\overset{\frown}{AC}$ is $\sqrt{3}$, find the length of EF.

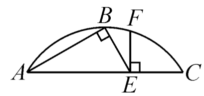


Fig.8

Team Round Answers

1. 44.

2. 180.

3. 2.

4. $\frac{25}{6}$.

5. 14.

6. $3\sqrt[3]{12}$.

7. $806\sqrt{5}$.

8. 15.

9. 25.

10. $\frac{2015}{1008}$.

11. 2.

12. 2.

13. 13.

14. $14 + 2\sqrt{3} + \sqrt{5}$.

15. 16.

16. 1343.

17. 3.

18. $\frac{3 + \sqrt{6} + \sqrt{15}}{2}$.

19. 4.

20. $\frac{\sqrt{11} - \sqrt{3}}{2}$.

Relay Round Problems

First Round

1A. If $\frac{a}{b}=\frac{c}{a}=2$, find $\frac{3a-c}{3b-d}$.

1B. Let $T=$ TR (The number you will Receive). Let a, b, and c be the sides that are opposite to angles A, B, and C, respectively, of $\triangle ABC$. If $\frac{a}{b}=\frac{b}{a+c}$, $\angle A=30°$, and $a=T$, find the area of $\triangle ABC$.

Second Round

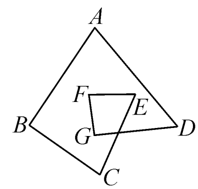


Fig.1

2A. Asshown in figure 1, let $\angle A+\angle B+\angle C+\angle D+\angle E+\angle F+\angle G=x°$. Find x.

2B. Let $T=$ TR. Suppose x and y are integer that satisfy the set of equations

$$\begin{cases} x+3y^2+2xy=18, & ① \\ y+3x^2+4xy=6, & ② \end{cases}$$

Find the value for $T(x+y)+2015$.

Third Round

3A. The cube of a natural number can be written as the sum of two or more consecutive odd numbers. For examples, $2^3=3+5$, $3^3=7+9+11$, and $4^3=13+15+17+19$. If 9^3 is written as the sum of two or more consecutive odd numbers, what is the largest odd number in this sum?

3B. Let $T=$ TR. Suppose p and q are non-zero natural numbers and that $p<q$. If $\frac{p}{q}=0.18\cdots$ and $q=110$, find $(p+T)$.

Relay Round Answers

1A. 2.

2A. 540.

3A. 89.

1B. $2\sqrt{3}$.

2B. 395 or 3455.

3B. 109.

Individual Round Problems

First Round

1. Suppose real number a and its reciprocal (multiplicative inverse) have the same value and real number b and its additive inverse have the same value, find all possible values for $a-b$.

2. Suppose real numbers x and y satisfying the equation $|x+y|+\sqrt{x-1}=0$. Find the value for $x^{2015}+y^{2016}$.

3. As shown in figure 1, quadrilateral $ABCD$ is inscribed in circle O. If $\angle DOB=100°$, find the degree measurement for $\angle BCD$.

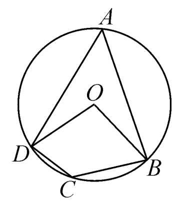


Fig.1

4. If the product of two 2-digit numbers $\overline{x2}$ and $\overline{2y}$ is 736, find $x+y$.

5. If the sum of all interior angles of a convex n-sided polygon is 7 times the sum of all its exterior angles, find the value of n.

6. Find the largest integer n such that $n^{300}<7^{200}$.

7. Suppose the graph of an inverse proportion function $y=\dfrac{k}{x}$ passes through point (1, 8). Let AB be the chord of Circle O with length k and the distance from center O to AB is 3. Find the radius of Circle O.

8. What is the probability of selecting any three consecutive primes from a list of prime numbers that are not larger than 50 so that the sum of these three number is not a prime?

9. How many values can x take on so that the points $M(1, 2)$, $N(6, 2)$, and $P(x, 0)$ form a right triangle $\triangle MNP$?

10. If the radius of the inscribed circle of a regular hexagon is 1, find the area of this hexagon's circumcircle.

11. Find the root(s) for equation $\left(x-\dfrac{1}{1-x}\right)^2\div\dfrac{x^2-x+1}{x^2-2x+1}=1$.

12. Rotate rectangle $ABCD$ clockwise $90°$ around point A to $AEFG$ position as shown in figure 2. If the area of $\triangle BCD$ is $6+2\sqrt{5}$, find the area of $\triangle DEF$.

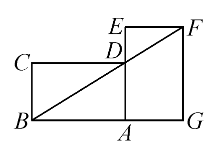
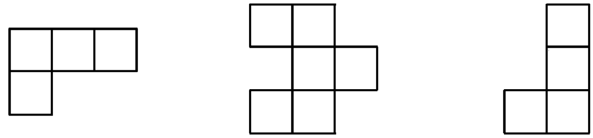


Fig.2

13. Suppose straight line $y=kx+b$ intersects hyperbola $y=\dfrac{k}{x}$ at points $A(x_1, y_1)$ and $B(x_2, y_2)$. Find the value of $x_1^3x_2+x_1^2+x_2^2+x_1x_2^3$.

14. A geometric solid is composed by pasting many identical small cubes together side to side. The figure 3 show the different views of this solid. Let m be the edge length of these cubes. Find the surface area of this solid.



Top View Front View Right Side View

Fig.3

Second Round

15. If $[2x+1]=3x-\frac{1}{2}$, find x.

(Note: $[x]$ represents the largest integer that is not greater than x.)

16. As shown in figure 4, the outside circle is the circumcircle of radius 2 of a regular hexagon. The shaded 6-pointed star region is formed by using each side of this hexagon as axis of symmetry and flip the corresponding arc $180°$. Find the proportion of the area of the shaded region to the area of the circumcircle.

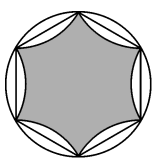


Fig.4

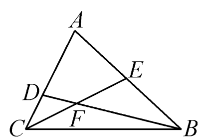


Fig.5

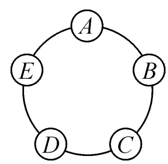


Fig.6

17. As shown in figure 5, points D and E are on sides AC and AB of $\triangle ABC$, respectively, and straight lines DB and EC intersect at point F. If the areas of $\triangle CDF$, $\triangle BEF$, and $AEFD$ are 3, 4, and $\frac{204}{13}$, respectively, find the area of $\triangle BCF$.

18. As shown in figure 6, five people A, B, C, D, and E are sitting around a circular table each holding 70, 62, 47, 67, and 54 marbles, respectively. How many times must they "adjust" so that each of them holds the same number of marbles? ("Adjust" once means one marble is transferred from one person to the person sitting next to him.)

19. As shown in figure 7, $\triangle ABC$ and $\triangle ACD$ are both equilateral triangles of side length 1. Fix the position of $\triangle ABC$ and rotate $\triangle ACD$ one revolution around $\triangle ABC$ using point C as anchor. Do the same thing using B and A as anchor. Find the total area that is swept by line segment AC after all three revolutions.

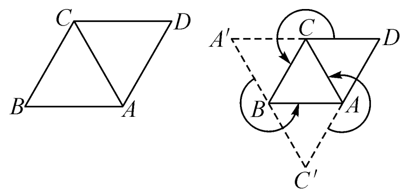


Fig.7

20. Suppose the parabola $y = -x^2 + bx$ passes through $B(2, 0)$. Let C be a point on the parabola's axis of symmetry and $A = (4, 4)$. Find the coordinates of C so that the length $AC + BC$ is smallest.

Individual Round Answers

1. ± 1.

2. 2.

3. $130°$.

4. 6.

5. 16.

6. 3.

7. 5.

8. $\frac{4}{13}$.

9. 4.

10. $\frac{4}{3}\pi$.

11. 0.

12. 4.

13. 0.

14. 30.

15. $\frac{5}{6}, \frac{7}{6}, \frac{3}{2}$.

16. $\sqrt{3} - 1$.

17. 5.

18. 23.

19. $\pi + \frac{\sqrt{3}}{2}$.

20. $(1, 1)$.